**Below is a concise documentation describing how each of the three retention models—Exponential, Power Law, and Gompertz—functions conceptually for analyzing player progression (or user retention). Citations for additional reading are provided at the end of each section.**

**1. Exponential Model**

**Overview**

The exponential model assumes that the fraction of players (or users) who remain at each successive “step” (e.g., game level) decreases at a roughly constant rate. Formally:



* p(Level) is the predicted retention fraction at a given level.
* A is the initial fraction (often A=1 if measuring from the very first level).
* r is the *per-level retention rate*, a constant between 0 and 1.

**Typical Usage in Retention Analysis**

* **Interpretation**: If r=0.95, then you lose 5% of players every level, on average.
* **Advantages**:
  + Very straightforward and easy to interpret.
  + Often a decent first approximation if churn is relatively stable across levels.
* **Limitations**:
  + Can be overly simplistic if the drop-off rate changes significantly at certain levels.
  + Does not capture “plateaus” or “acceleration” in churn very well.

**Reference**

* **Game Analytics: Maximizing the Value of Player Data** (Seif El-Nasr, Drachen, & Canossa, Springer, 2013) offers a broad overview of applying exponential decay models in user retention.
* **Nonlinear Regression Analysis and Its Applications** (Bates & Watts, Wiley, 1988) covers the mathematical background for fitting exponential functions.

**2. Power Law Model**

**Overview**

The power law model posits that retention follows a heavy-tailed distribution. A common form is:



* p(Level) is the predicted retention fraction at each level.
* a and b are parameters determined by fitting to observed data.
  + b often reflects how quickly retention drops.
  + a is a scaling constant.

**Typical Usage in Retention Analysis**

* **Interpretation**: A power law can capture scenarios where there’s a very steep drop early on, followed by a “long tail” of highly dedicated players who continue playing for many levels.
* **Advantages**:
  + Good for modeling “fat tail” distributions where a minority of users progress very far.
  + Can handle large variability in early vs. late-level behavior.
* **Limitations**:
  + Can be less intuitive than exponential models.
  + Some data sets do not exhibit truly power-law behavior; partial or hybrid models may fit better.

**Reference**

* **Power Laws, Pareto Distributions and Zipf’s Law** by M. E. J. Newman (*Contemporary Physics*, 46(5), 323–351, 2005) discusses the mathematical properties and real-world occurrences of power-law distributions.
* **Game Analytics: Maximizing the Value of Player Data** (Seif El-Nasr et al.) also touches on heavy-tailed user behavior and churn patterns.

**3. Gompertz Model**

**Overview**

Originally developed for actuarial and demographic studies, the Gompertz function is also used for survival analysis and can be adapted to player retention. One common form is:



* p(Level) is the fraction of players surviving (retained) up to that level.
* β and μ are parameters that shape the curve.
  + β typically controls how quickly churn accelerates or decelerates.
  + μ can shift the inflection point.

**Typical Usage in Retention Analysis**

* **Interpretation**: Gompertz curves can model situations where the churn rate changes over time (or level). For instance, if there is a slow churn initially but it accelerates (or decelerates) at higher levels, the Gompertz function can capture that dynamic more flexibly than a simple exponential.
* **Advantages**:
  + More nuanced than exponential, capturing different rates of change.
  + Common in “survival analysis,” which closely parallels player progression (players “survive” to higher levels).
* **Limitations**:
  + Requires iterative fitting (e.g., using specialized tools or solvers).
  + Interpreting β and μ can be less straightforward than the parameters of simpler models.

**Reference**

* **Survival Analysis: A Self-Learning Text** (Kleinbaum & Klein, Springer, 2012) explains the Gompertz function and related survival models.
* **Game Analytics: Maximizing the Value of Player Data** (Seif El-Nasr, Drachen, & Canossa) has chapters on advanced retention modeling that reference logistic-like functions such as Gompertz.

**Additional Notes**

* **Choosing the Right Model**: In practice, analysts often compare multiple models (exponential, power law, Gompertz, etc.) against actual data and select the one with the best predictive accuracy.
* **Hybrid or Piecewise Approaches**: Some games exhibit multiple “phases” of churn (e.g., a steep early drop-off and a slower late drop-off). In such cases, piecewise or hybrid models might outperform a single global curve.

**General References**

* **Game Analytics: Maximizing the Value of Player Data**, Eds. Magy Seif El-Nasr, Anders Drachen, and Alessandro Canossa, Springer, 2013.
* **Nonlinear Regression Analysis and Its Applications**, Douglas M. Bates & Donald G. Watts, Wiley, 1988.
* **Power Laws, Pareto Distributions and Zipf’s Law**, M. E. J. Newman, *Contemporary Physics*, 46(5), 323–351, 2005.
* **Survival Analysis: A Self-Learning Text**, David G. Kleinbaum & Mitchel Klein, Springer, 3rd ed., 2012.

These resources collectively provide the theoretical foundations and practical guidance for using exponential, power law, and Gompertz models in user retention and survival analysis.